

Queen Mary, University of London
BSc Examination by Course Units
MAS101 Calculus I

Wednesday, 3 May 2006, 2.30 pm

Duration: TWO HOURS

This paper has two Sections and you should attempt both Sections. Please read carefully the instructions given at the beginning of each Section.

Calculators are NOT permitted in this examination. The unauthorised use of a calculator constitutes an examination offence.

The examination questions begin overleaf. Please do not turn over until the examination commences.

SECTION A

Each question carries 8 marks. You should attempt ALL questions.

- A1. The functions $f(x)$ and $g(x)$ are defined for all real numbers x by:

$$f(x) = e^x, \quad g(x) = x^3 + \sin^2 x.$$

Calculate:

$$(a) \quad \frac{d}{dx}(f(x) \cdot g(x))$$

$$(b) \quad \frac{d}{dx}f(g(x))$$

- A2. Define what is meant by even and odd functions. Given that e^x is neither even nor odd, find the values of the constants a and b such that the function $F(x)$ given by

$$F(x) = ae^x + be^{-x}$$

is (i) odd, or (ii) even.

- A3. Evaluate the following limits

$$(a) \quad \lim_{x \rightarrow \infty} \frac{3x^{32} - 7x^{15} - 11}{8 - 2x^{19} - 12x^{32}}$$

$$(b) \quad \lim_{x \rightarrow 1} \frac{e^{x-1} - x}{(x-1)^2}$$

A4. Evaluate

$$\frac{d}{dx} \int_5^{\tan x} [t^4 + 2t^2 + 1] dt$$

without attempting to perform the integral. Simplify your result.

A5. (i) Find the equations for the tangent and the normal to the curve

$$y = e^x \sqrt{1-x}$$

at $x = 0$.

(ii) Test the following series for convergence or divergence. Give reasons for your answer.

$$\sum_{n=0}^{\infty} \frac{5n^5 - 11}{7 + 3n^5}$$

Section B questions continue overleaf

SECTION B

Each question carries 20 marks. You may attempt all questions. Except for the award of a bare pass, only marks for the best THREE questions will be counted.

B1. Evaluate the following integrals

$$(a) \quad \int_1^e x^3 \ln x dx$$

$$(b) \quad \int_0^{\pi/2} \sqrt{\sin x} \cdot \cos x dx$$

$$(c) \quad \int e^{3x} \sin 5x dx$$

B2. (i) What does it mean to say that the integral $\int_1^\infty f(x)dx$ converges? For what values of the constant $m \neq 1$ does the following integral

$$\int_1^\infty \left(\frac{1}{x^m} \right) dx$$

converge?

(ii) Derive the reduction formula for the integral

$$I_n = \int_0^{\pi/4} \tan^n \theta \, d\theta$$

in the form

$$I_n = \frac{1}{n-1} - I_{n-2}, \quad (n \geq 2).$$

Hence or otherwise, compute $\int_0^{\pi/4} \tan^8 \theta \, d\theta$.

B3. Sketch the curve

$$y = \frac{x^2}{\sqrt{2x+1}}.$$

Show, if they exist, all asymptotes, all maxima and minima and all inflection points.

B4. (i) State the de Moivre's Theorem for positive integers. Assuming this result, prove the de Moivre's Theorem for negative integers.

(ii) Use this theorem to express $\cos^5 \theta$ in terms of multiple angles.

(iii) Find all complex roots of the complex equation

$$z^7 + 1 + i = 0$$

[It is sufficient to leave the solutions to this part in polar form.]

B5. (i) Write down the n -th term of the power series

$$\frac{x}{1} + \frac{x^4}{2} + \frac{x^7}{6} + \frac{x^{10}}{24} + \dots$$

Hence determine the radius and interval of convergence of this power series giving reasons where needed.

(ii) Without doing any further calculations, state the radius of convergence of the power series

$$\frac{x}{2} - \frac{x^4}{3} + \frac{x^7}{7} - \frac{x^{10}}{25} + \dots$$

giving the reason for your answer.

(iii) Write down the first 3 non-zero terms of the Maclaurin series for the function

$$e^{5x}$$

Write down the error term involved in approximating the function by these 3 terms.

End of examination paper